Determination of Plate Efficiencies from Operational Data: Part II

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A method for the determination of the plate efficiencies for existing conventional and complex columns is presented. Information, such as any combination of product distributions, plate temperatures, and plate compositions, which is in addition to that required to solve a problem for a fixed column is used to determine efficiencies.

This paper has two primary objectives: the extension of previous methods (*8, 12*) for the determination of plate efficiencies from column performance data and the interpretation and presentation of the efficiencies determined for a variety of commercial columns.

The previous methods required that the operational data include enough information to form the specifications required by the Thiele-Geddes calculational procedure for predicting column performance. These methods used any additional information, including any number of plate temperatures and any number of product distributions, to determine a set of plate efficiencies. These methods were restricted to conventional columns.

The method described herein extends these basic methods to make use of any liquid or vapor compositions which may be known. In addition, the method is applicable to complex columns for which the compositions, flow rates, and thermal conditions of each side stream are known. The method involves the resolution of such columns into two or more simple, independent units which are referred to as *subunits*. After the column has been divided into subunits, the vaporization efficiencies for each subunit are determined independently by use of the method proposed by Taylor et al. (*12*) for conventional columns.

DIVISION OF A COLUMN INTO INDEPENDENT SUBUNITS

In general, the points of division of a column into subunits are at the plates where the composition of either the vapor or liquid is known. Since the composition of a side stream (vapor of liquid) is the same as that of the stream (vapor or liquid) leaving the plate from which the side stream is withdrawn, complex columns have points of division at each side stream.

To divide the column at plate j, it is necessary either to know T_j, T_{j+1}, and the composition of the liquid leaving plate j, or else the composition of the vapor leaving plate j, T_j, and T_{j-1}. The method is best illustrated by consideration of the complex column shown in Figure 1. Suppose that the performance data listed below are given for the column of Figure 1.

1. Composition, thermal condition, and flow rates F_1, F_2, and F_3 for the three feeds, and flow rates L_0, D, and W.
2. Number of plates, location of all feed plates, type (partial or total) of condenser, and column pressure.
3. Composition of the distillate, bottoms, and side stream, composition of the liquid leaving plate 7.
4. T_0, T_{N+1}, T_7, T_8, T_{12}, T_{13}, and any number of other temperatures.
5. The composition of the reflux (if the condenser is a partial condenser).

The column may be divided for calculational purposes into three subunits. The following equivalencies exist:

1. V_8 of Figure 1, V_{N+1} of subunit 1, and V_1 of subunit 2.
2. L_7 of Figure 1, L_N of subunit 1, and L_0 of subunit 2.
3. V_{13} of Figure 1, V_{N+1} of subunit 2, and V_1 of subunit 3.
4. L_{12} of Figure 1 and L_N of subunit 2.
5. $\bar{L}_{12}$ of Figure 1 and L_0 of subunit 3.
6. F_1 of Figure 1 and F of subunit 1.
7. F_2 of Figure 1 and F of subunit 2.
8. F_3 of Figure 1 and F of subunit 3.

The terminal flow rates of the subunits are calculated in a step-by-step manner commencing with the first subunit. Material balances and enthalpy balances are made for each of these subunits in a manner analogous to the procedure shown below for the first subunit. The material-balance and enthalpy-balance equations around the condenser are solved for the condenser duty which is given by

$$Q_c = L_0\,[H(x_0)_1 - h_0] + D\,[H(X_D)_1 - H_D] \qquad (1)$$

The material-balance and enthalpy-balance equations for

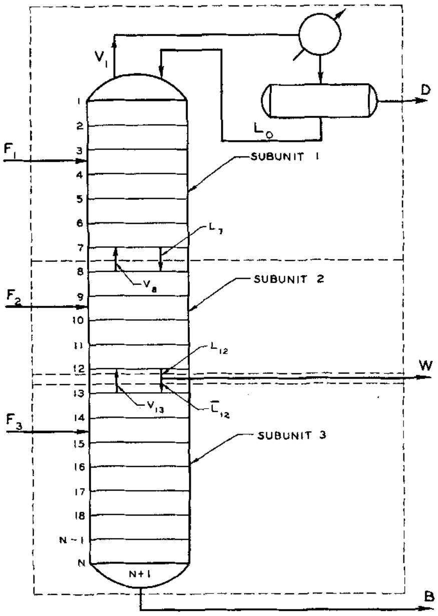


Fig. 1. Complex column with one side stream withdrawal.

the envelope enclosing subunit 1 (including the condenser) may be solved for L_N as follows:

$$L_N = \frac{F[H(X)_{N+1} - H] - D[H(X_D)_{N+1} - H_D] + Q_c}{[H(x_N)_{N+1} - h_N]} \tag{2}$$

The vapor rate V_{N+1} is given by

$$V_{N+1} = L_N + D - F \tag{3}$$

Since the x_{Ni}'s are known, the l_{Ni}'s are given by

$$l_{Ni} = L_N \, x_{Ni} \tag{4}$$

The $v_{N+1,i}$'s are then evaluated by material balance:

$$v_{N+1,i} = l_{Ni} + d_i - FX_i \tag{5}$$

After the calculations implied by Equations (1) through (5) have been performed, the terminal flow rates for subunit 1 will have been determined. Because of the equivalencies listed, L_o and V_1 of subunit 2 are known. The material-balance and enthalpy-balance equations for the envelope enclosing this second unit may be solved for L_N and V_{N+1} of subunit 2 in an analogous manner. Finally, the reboiler duty may be evaluated from an enthalpy balance around subunit 3.

DETERMINATION OF THE VAPORIZATION EFFICIENCIES OF SUBUNITS BY THE METHOD OF TAYLOR ET AL. (12)

The vaporization efficiency has been defined (8) by

$$E^{\circ}_{ji} = \frac{y_{ji}}{Y_{ji}} \tag{6}$$

where

$$Y_{ji} = K_{ji} \, x_{ji} \tag{7}$$

and K_{ji} is evaluated at the actual temperature of the liquid leaving plate j, and x_{ji} represents the actual mole fraction of component i in the liquid leaving plate j. If the liquid leaving plate j is at its bubble point, the sum of the Y_{ji}'s is unity; otherwise, the sum is greater than or less than unity depending upon whether the temperature of the liquid is above or below its bubble point temperature. Equations (6) and (7) are readily combined to give

$$y_{ji} = E^{\circ}_{ji} K_{ji} x_{ji} \tag{8}$$

Thus, E°_{ji} is seen to be precisely the multiplier required to obtain an equality between the actual composition y_{ji} of the vapor entering plate $j - 1$ and that calculated as Y_{ji} on the basis of the temperature and composition of the liquid leaving plate j. The quantity E°_{ji} bears a strong resemblance to the thermodynamic activity in that its definition is independent of the particular model proposed to describe the action that occurs on a plate. Although any number of models could be proposed for the determination of the vaporization efficiencies, only the one that follows in which the vaporization efficiency is assumed to be that product function given by

$$E^{\circ}_{ji} = \overline{E}^{\circ}_i \, \beta_j \tag{9}$$

has been investigated extensively.

According to the method proposed by Taylor et al. (12) for a single unit, a component efficiency $\overline{E}^{\circ}_i$ is determined for each known value of b_i/d_i. For each known temperature T_j a corresponding plate factor β_j is determined. For all components for which b_i/d_i is unknown, a single value of $\overline{E}_i^{\circ}$ (denoted by $\overline{E}_s^{\circ}$) is determined such that the sum of the distillate rates for these components is equal to that part of the specified value of D unaccounted for by the known values of b_i/d_i. For any plate for which T_j is unknown, Taylor et al. (12) suggest that the plate factor for this particular plate be taken equal to the one for some other plate and that the temperature for the particular plate be determined.

When this method is applied to each of the three subunits shown in Figure 1, it becomes possible to determine a component factor for each subunit, namely, $\overline{E}_i^{\circ 1}$, $\overline{E}_i^{\circ 2}$, and $\overline{E}_i^{\circ 3}$, where the second superscript denotes the number of the subunit. For these subunits, the θ_i functions are defined as follows:

$$\theta_i = \frac{(l_{Ni}/d_i)_{co}}{(l_{Ni}/d_i)_{ca}} \tag{10}$$

$$\theta_i = \frac{(l_{Ni}/v_{1i})_{co}}{(l_{Ni}/v_{1i})_{ca}} \tag{11}$$

$$\theta_i = \frac{(b_i/v_{1i})_{co}}{(b_i/v_{1i})_{ca}} \tag{12}$$

Also, the forms of the material-balance and enthalpy-balance equations for the three subunits differ from those for a conventional column. Each plate was closed by a component-material balance. This system of equations, tridiagonal in form, was solved for the component flow rates by Crout's method or the equivalent recursion formulas (6), which were first applied to distillation problems by Ball (2). Again, as found by Ball, round-off error was virtually eliminated.

The advantages of knowing intermediate plate compositions may be illustrated by considering the column shown in Figure 1 in another way. Suppose that $F_1 = F_3 = W = 0$. Then the column is reduced to a conventional column with a single feed stream, an overhead product stream, and a bottoms product. Suppose that the compositions of the liquid streams leaving plates 7 and 12 are again known. The column could again be resolved into the subunits shown in Figure 1.

Suppose that the feed contains five components. If the overhead product contained only components 1, 2, and 3 in measurable quantities, and the bottoms product contained only components 3, 4, and 5, then only one product distribution (b_3/d_3) would be specified. The method of reference 12 would yield only the single component efficiency $\overline{E}_3^{\circ}$ and the mean component efficiency $\overline{E}_s^{\circ}$,

TABLE 1. LIQUID COMPOSITIONS KNOWN FOR EXAMPLE 1

Plate No.	Methane	Ethane	Propane	*i*-butane	*n*-butane	*n*-pentane	Hexane	Heptane
0 (reflux)	0.0067	0.0641	0.7002	0.1600	0.0690	—	—	—
2	0.0012	0.0218	0.4340	0.2172	0.3258	—	—	—
6	0.0011	0.0173	0.2249	0.3102	0.4465	—	—	—
13	0.0007	0.0152	0.1901	0.2673	0.4361	0.0766	0.0107	0.0034
17	0.0001	0.00019	0.1190	0.1458	0.4374	0.2141	0.0610	0.0226
18	—	—	0.1000	0.1717	0.4415	0.2011	0.0686	0.0172
21	—	—	0.0359	0.1929	0.4724	0.2071	0.0678	0.0238
29 (reboiler)	—	—	0.0009	0.1251	0.3560	0.3310	0.1346	0.0524

TABLE 2. CALCULATED COMPONENT EFFICIENCIES FOR EXAMPLE 1

Zone of column	Component efficiency Methane	Ethane	Propane	*i*-butane	*n*-butane	Pentane	Hexane	Heptane
Condenser-plate 2	2.33	1.08	1.10	1.11	0.85	—	—	—
Plate 3-plate 6	2.22	1.05	1.08	0.94	1.07	—	—	—
Plate 7-plate 13	3.78	1.13	1.05	0.93	1.03	(1.01)	(1.01)	(1.01)
Plate 14-plate 17	3.06	2.72	1.05	1.03	0.95	0.98	1.20	2.04
Plate 18-plate 21	(1.00)	(1.00)	1.12	0.86	1.07	1.12	0.62	0.76
Plate 22-reboiler	—	—	1.14	0.95	1.05	1.01	1.11	1.34
Entire column by method of reference *12*	(1.11)	(1.11)	(1.11)	0.97	1.01	(1.11)	(1.11)	(1.11)

which would be used for each of the separated components.

Also, suppose that all five components were present in measurable quantities on plates 7 and 12. Then the analysis of subunit 1 would yield three component efficiencies, $\overline{E}_1^{o1}$, $\overline{E}_2^{o1}$, and $\overline{E}_3^{o1}$. As suggested by reference *12*, an $\overline{E}_s^{o}$ would be calculated for components 4 and 5.

The analysis of subunit 2 would yield a component efficiency for each component. The analysis of the bottom section of the column, subunit 3, would yield three component efficiencies, $\overline{E}_3^{o3}$, $\overline{E}_4^{o3}$, $\overline{E}_5^{o3}$. The component efficiency for each of the light components would be taken to be unity and the final value modified by the normalization factor.

Now consider the case where the concentration of the heaviest component, component 5, is negligible in the liquid leaving plate 7. Since F_1 of Figure 1 has been taken equal to zero in this discussion, Equation (2) becomes

$$L_N = \frac{Q_c - D\,[H(X_D)_{N+1} - H_D]}{[H(x_N)_{N+1} - h_N]} \qquad (13)$$

Because $X_{D,5}$, $x_{o,5}$, and $x_{N,5}$ are all approximately zero, the values of L_N, V_{N+1}, and Q_c are essentially independent of component 5. Therefore, a convention has been adapted which has the effect, in this case, of eliminating component 5 completely from the analysis of the top section of the column.

The important distinction between a concentration being negligible and merely unknown must be noted. Had $x_{N,4}$ and $x_{N,5}$ been unknown but not negligible, the solution of Equation (13) would have been impossible. Thus in order to divide a column at a plate, the concentrations of all components present in appreciable amounts must be known.

Illustrative Example 1

The application of the proposed method for the determination of the plate efficiencies for columns for which the composition of the liquid (or vapor) for one or more plates is known may be illustrated by use of the example presented by Gunness (*7*). A restatement of this example in terms of the present notation as well as its solution is presented elsewhere (*3*). The column described by Gunness had twenty-eight plates plus a reboiler and a partial condenser, and the temperature of each stage was known. The feed was composed of eight hydrocarbons ranging from methane through normal heptane. The product distributions b_i/d_i were known for only two components, *i*-butane and *n*-butane. Apparently, in the analysis of the samples, they were first cut into five fractions: ethane and lighter, propane, butanes, pentanes, and hexanes and heavier. Considerable inconsistencies arose when these fractions were further subdivided into their component parts. The ratio of *i*-butane to *n*-butane was somewhat inconsistent. The concentration profiles of these two components were erratic. The data listed in Table 1 are the compositions on plates where the *i*-butane and *n*-butane concentrations coincided with curves drawn through a plot of concentration versus plate number. Table 2 gives comparisons of the analyses by the methods of reference *12* and by the methods presented herein. The advantage of knowing intermediate plate compositions is evident from Table 2.

Knowledge of five plate compositions permitted computation of thirty-five component efficiencies. Only three component efficiencies could be evaluated when these additional specifications were not used. The values in Table 2 which are enclosed in parentheses are the mean component efficiencies evaluated for all the separated components. Fluctuations in the component efficiencies of say C_7 probably result from corresponding inaccuracies in the analyses.

For the entire column, the plate efficiencies were absorbed almost entirely into the component efficiencies, and further absorption of the plate efficiencies into the component efficiencies occurred as the column was divided into subunits. For the entire column, the plate factors were close to unity ($0.96 \leqq \beta_j \leqq 1.04$), and when the column was divided into subunits, the plate factors were still closer to unity.

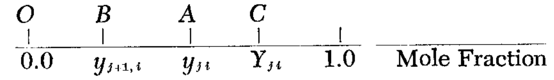
PHYSICAL SIGNIFICANCE OF THE VAPORIZATION EFFICIENCIES

The physical significance that may be associated with the component efficiencies $\overline{E}_i$ and plate factors β_j is useful as an aid in the interpretation of the results obtained for commercial columns in different types of service.

This analysis is initiated by the comparison of the vaporization efficiency with the modified Murphree efficiency, defined by

$$E^M_{ji} = \frac{y_{ji} - y_{j+1,i}}{Y_{ji} - y_{j+1,i}} \qquad (14)$$

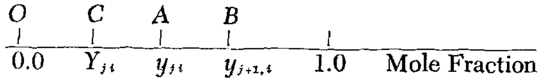
Consider some component i which is being enriched in the vapor phase as the vapor rises through some section of the column. Since component i is being enriched, $y_{ji} > y_{j+1,i}$. Suppose that $E^M_{ji} < 1.0$, and let y_{ji} and $y_{j+1,i}$ be located at A and B, respectively, on the number line below. Since $E^M_{ji} < 1.0$, it follows from Equation (14) that Y_{ji} is at some point C to the right of A. The modified Murphree efficiency is $\overline{BA}/\overline{BC}$, and the vaporization efficiency is $\overline{OA}/\overline{OC}$. Thus, a component



which is being enriched in the vapor and has a modified

Murphree efficiency less than unity has a vaporization efficiency that is less than unity.

Next, consider a component whose concentration in the vapor phase is being diminished as the vapor rises through the column. Then, $y_{j+1,i} > y_{ji}$, and the corresponding point A is to the left of B on the number line shown below. If the modified Murphree efficiency is less than unity, then it follows from Equation (14) that Y_{ji} is at some point C to the left of point A. The modified Murphree efficiency is $\overline{AB}/\overline{CB}$, and the vaporization efficiency is $\overline{OA}/\overline{OC}$. Thus, a component whose concentration is being diminished has a vaporization efficiency greater than unity if its modified Murphree efficiency is less than unity.



Thus, when the modified Murphree efficiency is less than unity, the following conclusions can be drawn. The mole fractions of the relatively light components in the vapor increase, on the average, as the vapor passes up through the column, and hence values of the E°_{ji} less than unity may be expected for these components. Similarly, values of E°_{ji} greater than unity are to be expected for the relatively heavy components, since their concentrations are, on the average, diminished as the vapor rises through the column.

From this interpretation of the E°_{ji}'s, reasonable physical explanations for the values obtained for the $\overline{E}_i^\circ$'s and the β_j's may be inferred. In view of the fact that the degree of separation is independent of the plate factors at total reflux (*3*, *12*), it may be assumed that most of the efficiency will be absorbed in the $\overline{E}_i^\circ$'s even at operating refluxes. The values of the β_j's may be expected to vary about the value unity.

In view of the fact that the E°_{ji}'s for the relatively light components can be expected to be less than unity and those for the relatively heavy components can be expected to be greater than unity, the $\overline{E}_i^\circ$'s for the lights may be expected to be less than the $\overline{E}_i^\circ$'s for the heavies. Thus, in the common situation in which less separation is achieved on an actual plate than would be achieved on a theoretical plate, $\overline{E}_i^\circ$ would likely be a decreasing function of the volatility. This characteristic is illustrated by the following example.

Illustrative Example 2

The data for this example were collected by Esso Research and Engineering Company (*4*). A complete statement of this example is given by Davis (*3*) (see Example 1 of this reference). The column contained thirty plates plus a condenser and a reboiler, and five temperatures were known. The feed contained nine hydrocarbons, ranging from methane through pentene-1, and the b_i/d_i's were known for propane, *i*-butane, *n*-butane, and butene-1. The calculated component efficiencies increased with volatility as predicted by the above analysis. The component efficiencies were propane, $\overline{E}_i^\circ = 0.96$; *i*-butane, $\overline{E}_i^\circ = 1.03$; *n*-butane, $\overline{E}_i^\circ = 1.41$; and pentene-1, $\overline{E}_i^\circ = 2.85$. The plate factors were assigned in accordance with the method given by reference *13*. The plate factors had the following values: $\beta_0 = 0.99$, $\beta_1 = 1.00$, $\beta_6 = 1.07$, $\beta_{30} = 0.93$, and $\beta_{31} = 0.92$.

When the equilibrium data are not accurate, the $\overline{E}_i^\circ$'s and the β_j's reflect these inaccuracies as well as the departure of the actual plates from perfect plates. If the activity correction is not included in the equilibrium constant K_{ji}, then

$$Y_{ji} = \gamma_{ji} K_{ji} x_{ji} \tag{15}$$

and the expression for y_{ji} corresponding to Equation (8) is

$$y_{ji} = E^\circ_{ji} \gamma_{ji} K_{ji} x_{ji} \tag{16}$$

If the activity correction is not included in the equilibrium constant but Equation (7) is used rather than Equation (15), then the activity corrections are absorbed in the vaporization efficiencies. In such cases each $\overline{E}_i^\circ$ could be expected to take on a value which reflects the mean of the activity coefficient for component i over all j. The β_j's could be expected to take on values which reflect the activity corrections not absorbed in the component efficiencies. The effect of the activity coefficients on the plate factors is demonstrated by the example that follows.

Illustrative Example 3

This example, provided by a chemical company, had one component (called D) that exhibited an appreciable activity coefficient. The activity coefficient for component D varied significantly throughout the column, while the activity coefficient of each of the remaining fifteen components was approximately equal to unity. The column contained fifty-five plates plus the condenser and reboiler, and eleven temperatures and eight b_i/d_i's were known. A complete statement of this example is presented by Davis (*3*) (see Example 2 of this reference). The performance data were analyzed in two ways. In the first analysis no activity correction was made, and the equilibrium constants were taken to be of the form

$$K_{ji} = \left[\frac{P_i^\circ}{P_b}\right]\left[\frac{P_b}{P_j}\right] \tag{17}$$

where the cube root of $(P_i^\circ/P_b)/T$ was curve fitted to a polynomial of degree three in T. In the second analysis, the activity coefficients were given by the Redlich-Kister model, Equation (5) of reference *1*.

The results of the two analyses showed that the activity correction for component D was absorbed partially by the component efficiency for component D and partially by the plate factors. Figure 2 shows the activity coefficients for component D calculated in the second analysis of the data. The activity coefficient passed through a maximum of 6.18 in the top section of the column. The plate factors calculated with no activity corrections reflect this maximum as is evident from Figure 2. The principal part of the activity correction was absorbed in the component efficiency for component D. The use of the activity corrections reduced the value of $\overline{E}_i^\circ$ for component D from 4.81 to 1.41. Figure 2 shows that the plate factors were much closer to unity when the corrections were used.

RELATIONSHIP BETWEEN COMPONENT AND OVERALL EFFICIENCIES

Although the relationship between these two types of efficiencies is exact only for columns operating at total reflux (or recycle), it provides a meaningful way to present the component efficiencies for columns operating at refluxes other than total. The development of Equation (25) follows.

As shown in reference *13*, the product distribution for a column at total reflux is given by

$$\frac{b_i}{d_i} = \frac{B/D}{\left(\prod_{j=0}^{N+1} K_{ji}\right)(\overline{E}_i^\circ)^{N+2}} \tag{18}$$

where it is understood that the β_j's are selected or normalized such that their product over all j has the value of unity. Also, in the development of this equation it is supposed that the column possesses a partial condenser. For

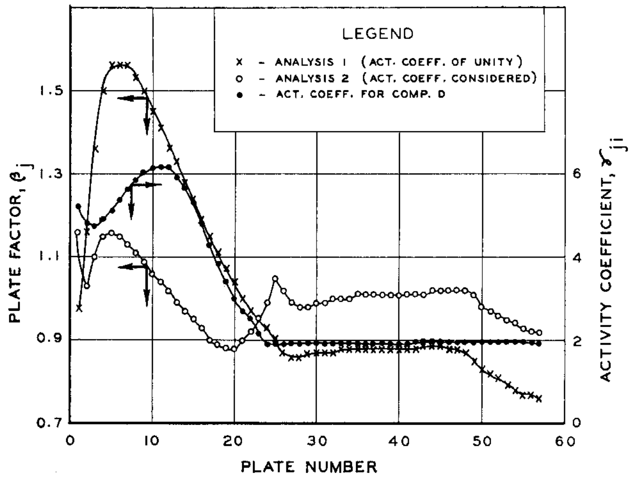


Fig. 2. Comparison of plate factors example 3.

a column with a total condenser, the product of the K_{ji}'s in Equation (18) is from $j = 1$ to $j = N + 1$ and the power to which $\overline{E}_i^{\,o}$ is raised is $N + 1$.

Winn (*14*) pointed out that the K values for hydrocarbons obey the relationship

$$K_{ji} = a_i K_{jb}^{\,c_i} \tag{19}$$

over wide ranges of temperature at a given pressure. The quantities a_i and c_i depend upon only the identity of the component at any given pressure. When this expression for K_{ji} is used in Equation (19), the following result is obtained:

$$\frac{b_i}{d_i} = \frac{B/D}{\left(\prod_{j=0}^{N+1} K_{jb}^{\,c_i}\right)(a_i \overline{E}_i^{\,o})^{N+2}} \tag{20}$$

If in Equation (19) the subscript i is set equal to b (the base component) and then both sides of the resulting equation are raised to the c_i power, the following result is obtained:

$$\left[\frac{b_b}{d_b}\right]^{c_i} = \frac{(B/D)^{c_i}}{\left(\prod_{j=0}^{N+1} K_{jb}^{\,c_i}\right)(\overline{E}_b^{\,o})^{c_i(N+2)}} \tag{21}$$

Eliminating the product involving the K_{jb}'s from Equations (20) and (21) and then solving the expression so obtained for the actual number of plates one gets

$$(N+2)_{\text{act}} = \frac{\log_e \left[\dfrac{[(b_b/d_b)/(B/D)]^{c_i}}{(b_i/d_i)/(B/D)}\right]}{\log_e \left[\dfrac{a_i \overline{E}_i^{\,o}}{(\overline{E}_b^{\,o})^{c_i}}\right]} \tag{22}$$

Consider a hypothetical column composed of perfect plates (all of the vapor leaving a plate is in equilibrium with the liquid on the plate). For such a column, $E^o_{ji} = \overline{E}_i^{\,o} = 1$ for all i and j, and Equation (22) becomes

$$(N+2)_{\text{theo}} = \frac{\log_e \left[\dfrac{[(b_b/d_b)/(B/D)]^{c_i}}{(b_i/d_i)/(B/D)}\right]}{\log_e \; a_i} \tag{23}$$

The hypothetical column might be further defined as one which has that number of plates necessary to give the same set of product distributions b_i/d_i as those observed for the actual column. However, in general there is no single hypothetical column such that $N + 2$ as computed by Equation (23) is independent of i unless, of course, the actual column also possesses perfect plates.

An overall efficiency E_i may be defined as the number of theoretical plates (required to obtain the same product distributions for two components as those obtained in the actual column) divided by the number of actual plates. The following expression for the overall efficiency E_i for any pair of components i and b is obtained by dividing corresponding members of Equation (23) by those of Equation (22):

$$E_i = \frac{(N+2)_{\text{theo}}}{(N+2)_{\text{act}}} = 1 + \frac{\log_e \dfrac{\overline{E}_i^{\,o}}{(\overline{E}_b^{\,o})^{c_i}}}{\log_e \; a_i} \tag{24}$$

Therefore

$$\log_e \overline{E}_i^{\,o}/(\overline{E}_b^{\,o})^{c_i} = (E_i - 1) \log_e a_i \tag{25}$$

Although this expression is exact only at total reflux, the component efficiencies for a column at any operating reflux may be displayed graphically by plotting $\log_e \overline{E}_i^{\,o}/(\overline{E}_b^{\,o})^{c_i}$ vs. $\log_e a_i$. The slope of this curve at any point is not only a good measure of the overall efficiency (at total reflux, the slope is exactly equal to $E_i - 1$) but also a good indicator of column performance. In general the departure of the curve for such a plot from the horizontal [the line $\overline{E}_i^{\,o}/(\overline{E}_b^{\,o})^{c_i} = 1$] is a measure of the inefficiency of the column operation. For a column with perfect plates, $\overline{E}_i^{\,o} = \overline{E}_b^{\,o} = 1$ for all i. The more volatile a component is, the larger the value of the corresponding a_i. In the discussion of the vaporization efficiency, the conclusion was drawn that for columns composed of plates that are less efficient than perfect plates, the component efficiency may be expected to be a decreasing function of component volatility. Thus, if the quantity $\overline{E}_b^{\,o}$ is approximately unity, the log-log plot of $\overline{E}_i^{\,o}/(\overline{E}_b^{\,o})^{c_i}$ vs. a_i may be expected to have a negative slope for columns composed of plates that are less efficient than perfect plates.

Vaporization Efficiencies for Several Depropanizers and Other Columns

The performance of columns with various types of trays are summarized in reference *4*. Furthermore, additional data have been made available by the Humble Oil and Refining Company (*9*). The performance data were analyzed, and vaporization efficiencies were evaluated. In Figure 3 the results are given for two bubble-cap columns and a jet-tray column which were in similar, depropanizer service. Separate curves are drawn for the light components ($a_i > 1$) and the heavy components ($a_i < 1$). A further refinement could be obtained by use

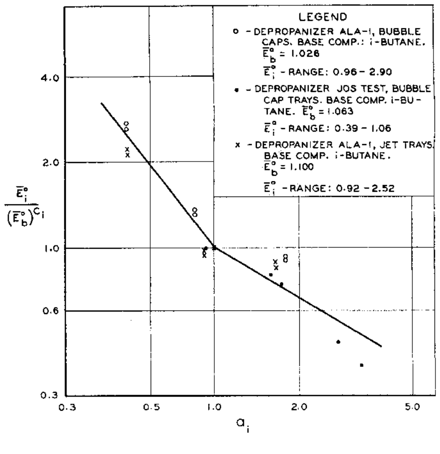


Fig. 3. Efficiencies for columns with bubble cap and jet trays.

of separate curves for the individual columns. The performance data analyzed were for Tests F-1, F-2, I-3, and I-5 of reference *4* and the JOS Test of reference *9*. Complete statements of all examples considered in this section are given by either Taylor (*13*) or Davis (*3*).

Figure 4 gives the results of the analyses of the performance data of three columns with sieve trays and a column with ripple trays. Two of these columns were in depropanizer service similar to that of the bubble-cap columns and jet-tray column. The results of these analyses fit a single straight line quite closely. This line is much flatter than that of Figure 3, indicating more efficient operation. This conclusion confirms that expressed in reference *4* that the overall efficiencies of the columns with ripple trays and sieve trays were higher than those equipped with jet trays and bubble-cap trays at the operating conditions at which the data were available. Figure 4 is based on the analysis of ten sets of performance data from references *4* and *9*. Fewer points are indicated on Figure 4 than might be expected from this number of analyses because some of the points in Figure 4 represent multiple incidences.

The overall efficiency of columns at operating refluxes is reflected in the plate factors. Reasoning similar to that used in interpreting the significance of the component efficiencies is used. Since the light components predominate in the top section and the heavy components predominate in the bottom section of the column, values of the plate factors that are larger for the top section than for the bottom section have the effect of making the light components appear more volatile and the heavy components appear less volatile. This may be interpreted as an increased overall efficiency. Relatively smaller values for the plate factors for the top section have the effect of making the light components appear less volatile and the heavies appear more volatile. This may be interpreted as a lower overall efficiency.

The plate factors obtained for the analyses whose component efficiencies are represented in Figures 3 and 4 were quite close to unity except for those for the JOS Test. For all others the plate factors, which are tabulated in reference 3, were generally between 1.05 and 0.95 with the exception of an isolated plate factor of 1.1 and one of 0.91. For the JOS Test the component efficiencies indicated an efficiency of the order obtained with the other bubble-cap column and with the jet-tray column. The plate factors obtained for the JOS Test varied approximately linearly from 1.4 at the top of the column to 0.81 at the bottom; thus, the plate factors for the top section are larger than those for the bottom section of the column. This would lead to an increased overall efficiency. This effect is, however, strongly overridden by the effect of the component efficiencies, and the conclusion drawn above concerning the relative efficiencies of the various columns is not materially affected.

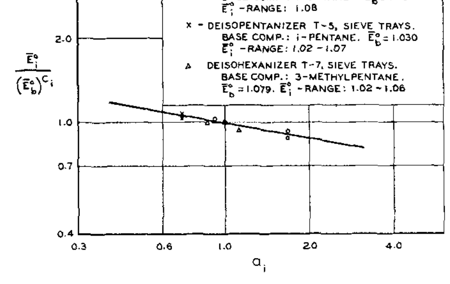


Fig. 4. Efficiencies for columns with sieve and ripple trays.

The discussion above relates the vaporization efficiencies to the familiar overall efficiency, but it should be pointed out that the evaluation of vaporization efficiencies by the methods presented here and in reference *12* is more than just a means of getting a measure of the overall efficiency. If appropriate values of the vaporization efficiency for each component on each plate were available, the exact performance of the column with respect to product distributions, plate temperatures, liquid and vapor flow rates, and plate compositions could be determined mathematically. The methods given here may be used to evaluate sets of vaporization efficiencies which satisfy all such knowledge that is available about a column.

The results presented above came from analyses of various types of performance data. Some of the runs were designed to evaluate overall efficiencies; others were field tests such as acceptance tests. The results must be viewed with this in mind. Furthermore, column performance may not conform precisely to the simplified model in which the vaporization efficiency is the product function given in Equation (9). However, the data from a number of runs in which the objective was to determine column efficiencies at total reflux were available (*5*), and in these cases the simplified product model for the vaporization efficiency represents the column operation quite well. This was illustrated by Example 2 of reference *12*, in which the calculated compositions of the liquids on intermediate plates of the column were in substantial agreement with the observed compositions.

Data were available for two series of runs that were made on a column equipped with ballast (valve) trays and operated at total reflux (*5*). In the first series of runs, a ternary system of propane, *i*-butane, and *n*-butane was used; in the second, a cyclohexane—*n*-heptane binary was used. The results of the analyses of these data are given by Taylor (*13*) and Davis (*3*). The performance data for seventeen tests that included intermediate compositions and the subunits were analyzed independently. Since the model chosen fitted the overall column operation quite well, essentially the same vaporization efficiencies resulted whether the column performance was analyzed as a whole or by subunits. Division of the column into subunits resulted in separate component efficiencies for the subunits and plate factors which were essentially unity for each plate. Component efficiencies differing from but close to unity were obtained for the individual columns. However, when the results of all tests were considered together, no real significant trend from perfect plates was observed. For the most part, the influence of the plate factors as discussed above ran counter to that of the component efficiencies. The plate factors were quite close to unity, ranging entirely between 0.95 and 1.05. Again, the influence of the component efficiencies outweighed that of the plate factors insofar as the overall efficiency was concerned, and the overall efficiency appeared to be slightly greater than unity for some runs and slightly less than unity for others. However, it is again emphasized that the methods presented herein are not efforts to evaluate overall efficiencies. In the total reflux cases cited above, the overall efficiency was quite close to unity, but, if the vaporization efficiency for each component had been taken equal to unity on each plate, neither the product distributions nor the plate temperatures would have been duplicated as they were by the calculated vaporization efficiencies.

Vaporization Efficiencies of Absorbers

The performance data of a number of absorbers were available (*11*). These data were collected independently at different locations by ten different companies. Cer-

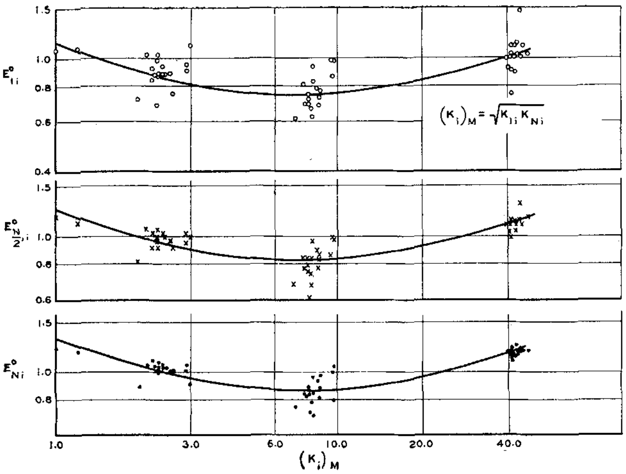


Fig. 5. Vaporization efficiencies determined from nineteen runs for six different absorbers at 60 lb./sq.in.abs.

tainly, the accuracy and particularly the consistency of these data could not be expected to compare with data collected in a single location at the controlled conditions of a laboratory. With this in mind and the fact that these fields tests did represent the best data available, these data were analyzed with the hope of demonstrating the general behavior of these units.

Absorbers, which have the configuration of the subunit 2 of Figure 1 with the value of F equal to zero, present a special case. The top feed L_o is commonly a liquid below its bubble point, and the bottom feed V_{N+1} is a vapor above its dew point. The net direction of mass transfer of the components in the rich gas is from the vapor phase to the liquid phase on most plates of the absorber. In a conventional column the net direction of transfer of such light components is often from the liquid phase to the vapor phase on most plates of the column. As a result deviation from equilibrium in absorbers is reflected in a different way by the vaporization efficiencies. If there is less transfer between phases on the plates of an absorber than would occur on equilibrium plates, the liquid remains leaner in the light components than would be predicted by the equilibrium expression $y_{ji} = K_{ji}x_{ji}$. Inefficiency in absorbers is indicated by a value larger than unity for the multiplier E°_{ji} in the expression, $y_{ji} = E^{\circ}_{ji}K_{ji}x_{ji}$.

The only temperatures that were known were the temperatures of the terminal streams. The compositions of the rich gas and lean gas were given, but the concentrations of the components lighter than methane were not well enough known so that their product distributions could be specified. Also, the concentrations in the lean gas of the components heavier than propane were seldom well enough known that their product distributions could be specified. The average molecular weight of the lean oil was specified. The lean oil was taken to be a single component free of any of the light components present in the rich gas.

The procedure used for the determination of efficiencies in absorbers consisted of a modified version of the one proposed by Taylor et al. (*12*). Since only two temperatures T_1 and T_N were known, only two β's could be determined by Taylor's procedure. When the procedure of Taylor et al. was followed, the remaining plates in the absorber were assigned β's equal to either β_1 or β_N. However, the conditions on the terminal plates differed widely from each other and from those on the remaining plates, and β_1 and β_N determined by this scheme differed from each other appreciably. On the other hand, the temperatures of the top and bottom plates were only a few degrees apart. In view of this it was concluded that in this instance temperatures could be assigned more accurately than could plate factors, and for absorbers Taylor's procedure was so modified. Although other temperature profiles could have been selected, a linear one between T_1 and T_N was chosen because these temperatures were relatively close together.

Four groups of absorber data were analyzed. The data were taken from absorbers operated at 60, 200, 480, and 1,000 lb./sq. in. abs. The plate factors were quite close to unity in nearly all cases. In particular, the plate factors were essentially unity for all of the interior plates for most columns. The vaporization efficiencies varied very little with plate number for all of the interior plates. The vaporization efficiencies for the distributed components on the top, middle, and bottom plates calculated for nineteen sets of data from six different absorbers at an operating pressure of 60 lb./sq. in. abs. are plotted against the mean equilibrium constants for these components in Figure 5. The vaporization efficiencies for the other interior plates are quite close to those plotted for the middle plate. The right-most clusters are for the component methane, the central clusters are for ethane, and the left clusters are for propane.

Similarly, the vaporization efficiencies were determined for six sets of performance data from five separate absorbers at an operating pressure of 200 lb./sq. in. abs. As shown by Davis (*3*), the vaporization efficiencies were quite similar to those obtained for the lower pressure absorbers. The vaporization efficiencies for methane were greater than unity, those for ethane were les than unity, and those for propane varied about the value of unity.

For the anomalous results obtained for ethane, there exist perhaps several explanations. For example, if the K values for ethane were too high relative to the other components, curves of the shape shown in Figure 5 would be expected. On the other hand, curves of this same shape could be produced by errors in the analyses whereby the relative amounts of ethane were too high in the rich oil and too low in the stripped gas. However, the latter explanation does not appear too plausible because the data for the 60 and 200 lb./sq. in. abs. absorbers were collected at different locations by ten different companies.

The vaporization efficiencies corresponding to fifteen sets of performance data taken from three different absorbers operating at 480 lb./sq. in. abs. were determined. A graph of these results is given by Davis (*3*). A more or less steady rise of the vaporization efficiency was observed for the three components propane, ethane, and methane. Similar results were obtained for absorbers operating at 1,000 lb./sq. in. abs. as shown in Figure 6.

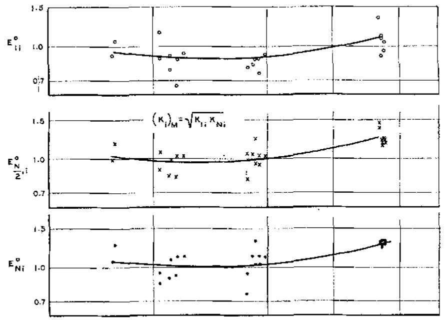


Fig. 6. Vaporization efficiencies determined from eight runs for seven different absorbers at 1,000 lb./sq.in.abs.

The efficiencies at this pressure were based on fifteen sets of performance data taken from three different absorbers.

The method presented herein together with the one by Taylor et al. (*12*) makes it possible for the industry to obtain almost perfect models of their actual columns through the use of efficiencies determined from the operational data for the respective units.

CONCLUSIONS

For any combination of product distribution, plate temperatures, and plate compositions, the corresponding plate efficiencies may be determined by the method presented.

Column performance is vividly reflected by the vaporization efficiencies ($E^{o}_{ji} = \overline{E}^{o}_{i}\beta_j$) and in particular by the component efficiencies. Poor column performance (poor separation) is reflected by values of the component efficiency $\overline{E}^{o}_{i}$ less than unity for relatively light components and by values of $\overline{E}^{o}_{i}$ greater than unity for relatively heavy components. That is, the effective K value ($E^{o}_{ji}K_{ji}$) of the relatively light components is diminished, and consequently these components appear to be heavier than they are. Similarly, the relatively heavy components are made to appear lighter than they are by the increase of their effective K values.

For conventional distillation columns in hydrocarbon service, the plate factors β_j can be expected to hover closer to unity than the component efficiencies $\overline{E}^{o}_{i}$. If the activity coefficient for one or more components deviates widely from unity in the same direction and this is not included in the equilibrium relationships used in the determination of the plate efficiencies, this deviation will be reflected by the plate factors as demonstrated in Figure 2. From the results of the Gunness example and others, it appears that as the number of subunits of a given column is increased, the proportion of the plate efficiency E^{o}_{ji} accounted for by the component efficiencies $\overline{E}^{o}_{i}$ increases, and the plate factors tend toward unity. Also, the difference in the operating conditions of different plates of a column such as the top and bottom plates of an absorber is uniquely reflected by the vaporization efficiencies. Finally, efficiencies determined by the proposed method may be of immediate use in the improvement of the models of existing columns and in the design of future columns.

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NOTATION

B = molal flow rate of bottoms product
b_i = molal flow rate of component i in the bottoms
c = number of components in the feed
d_i = molal flow rate of component i in the distillate
D = molal flow of distillate
E^{o}_{ji} = vaporization efficiency, defined by Equation (6)
$\overline{E}^{o}_{i}$ = component efficiency
F = molal flow rate of feed
h_{ji} = molal enthalpy of component i in the liquid leaving plate i
H_{ji} = molal enthalpy of component i in the vapor leaving plate j
H_i = molal enthalpy of component i in the feed, regardless of phase
H_{Di} = molal enthalpy of component i in the distillate, regardless of phase
$H(x_j)_{j+1}$ = enthalpy of a stream evaluated on the basis of the vapor enthalpies at the temperature of plate $j + 1$ and the liquid composition of plate j
K_{ji} = equilibrium constant for component i on plate j
l_{ji} = molal flow rate of component i in the liquid leaving plate j
L_j = molal flow rate of liquid leaving plate j
$\overline{L}_j$ = molal flow rate of liquid entering plate $j + 1$
N = number of plates in the column
P^{o}_{i} = vapor pressure of component i
P_b = pressure used in evaluating constants in Equation (17)
P_j = pressure on plate j
Q_c = condenser duty
T_j = temperature on plate j
v_{ji} = molal flow rate of component i in the vapor leaving plate j
V_j = molal flow rate of vapor leaving plate j
W = molal flow rate of side withdrawal stream
x_{ji} = mole fraction of component i in the liquid leaving plate j
X_i = mole fraction of component i in the feed
X_{Di} = mole fraction of component i in the distillate
y_{ji} = mole fraction of component i in the vapor leaving plate j
Y_{ji} = product defined by Equation (7) or (15)
β_j = plate factor
γ_{ji} = activity coefficient of component i on plate j
θ_i = function defined by Equations (10), (11), and (12)

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